

1 **Methods for Quantitative Risk Analysis for Travel Demand Model Forecasts**

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3  
4 Thomas Adler\*  
5 RSG  
6 55 Railroad Row  
7 White River Junction, VT 05001  
8 802.295.4999  
9 tadler@rsginc.com

10  
11 Michael Doherty  
12 URS  
13 1625 Summit Lake Drive  
14 Tallahassee, FL 32317  
15 850.574.3197  
16 michael.doherty@urs.com

17  
18 Jack Klodzinski  
19 Florida's Turnpike/URS  
20 Florida's Turnpike Headquarters  
21 Ocoee, FL 34761  
22 407.264.3819  
23 jack.klodzinski@dot.state.fl.us

24  
25 Raymond Tillman  
26 RT Consultancy  
27 345 West 58th Street  
28 New York, NY 10019  
29 212.315.3566  
30 raymond.tillman@lmstone.com

31  
32  
33 \*Corresponding Author

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## 1 Abstract

2 Travel demand forecasting models have played a critical role in transportation planning, supporting  
3 the evaluation of policies, programs and projects that involve complex interactions between the activity  
4 system and the transportation system. Both the state-of-the-art and the state-of-the-practice in travel  
5 demand modeling have advanced considerably over the many decades since the original four-step model  
6 structure was conceived. However, the models are not now, and never will be, perfect representations of  
7 the systems they represent and so there are inevitably uncertainties around the forecasts that these models  
8 generate. There are many applications in which the travel demand forecasts are important, for example, in  
9 determining whether a given alternative is financially or technically feasible or meets some benefit  
10 threshold. In these applications, uncertainties in the model forecasts may translate directly into risks of  
11 not accomplishing the objectives related to the decision to implement or not implement the alternative.  
12 For projects that involve outside financing, this threshold varies greatly between equity and lender  
13 participants because of their differing risk-reward profiles.

14 Several previous papers and reports have described the uncertainties associated travel demand  
15 forecasting and recommended ways of improving the state-of-the-practice. Among those  
16 recommendations is the application of formal quantitative risk analysis methods. This paper summarizes  
17 the existing literature and describes the application of one relatively straightforward but robust approach  
18 for conducting quantitative risk analysis with travel demand forecasting models.

19 The formal risk analysis approach described here can assist by providing a more complete evaluation  
20 of a project's likelihood of achieving specified objectives. In addition, it can have a broader application  
21 in the development of traffic and revenue forecasts other than a "most likely" or 50% probability of  
22 attainment ("P50") scenario. When another probability level or target is requested (e.g., 75% by a debt  
23 provider or rating agency), traffic and revenue analysts can use expected values for many required inputs  
24 and prepare a P50 forecast. They can then apply the procedures outlined here to determine the  
25 probabilities of attaining other levels of demand or revenue.  
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## 1 INTRODUCTION

2 Travel demand forecasting models have played a critical role in transportation planning, supporting  
3 the evaluation of policies, programs and projects that involve complex interactions between the activity  
4 system and the transportation system (see, for example, *1*). Both the state-of-the-art and the state-of-the-  
5 practice of travel demand modeling have advanced considerably over the many decades since the original  
6 four-step model structure was conceived. However, the models are not now, and never will be, perfect  
7 representations of the road or transit systems they represent and so there are inevitably uncertainties  
8 around the forecasts that these models generate.

9 In some travel demand modeling applications, such as those involving comparisons among project  
10 alternatives, uncertainties in the absolute magnitudes of the forecasts may not be as important. For  
11 example, the decision to be supported by travel demand forecasts may involve the relative position or  
12 ranking of alternatives, one or more of which will be selected regardless of magnitude in the demand.  
13 However, there are many other applications in which the absolute magnitudes of the forecasts are  
14 important, for example, in determining whether a given alternative is financially or technically feasible or  
15 meets some benefit threshold. With these applications, uncertainties in the model forecasts may translate  
16 directly into risks of not accomplishing the objectives related to the decision to implement or not  
17 implement the alternative. There are clearly many transportation planning applications of travel demand  
18 models that fall into this latter category such as tolled express lanes for managed lanes projects.

19 Several previous papers and reports have described the uncertainties associated with travel demand  
20 forecasting and recommended ways of improving the state-of-the-practice. Among those  
21 recommendations is the application of formal quantitative risk analysis methods. This paper summarizes  
22 existing literature and describes the application of one relatively straightforward but robust approach for  
23 conducting quantitative risk analysis with travel demand forecasting models.

24 The formal risk analysis approach described here can assist by providing a more complete evaluation  
25 of a project's likelihood of achieving specified objectives. In addition, it can have a broader application  
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27 attainment ("P50") scenario. When another probability level or target is requested (e.g., 75% by a debt  
28 provider or rating agency), traffic and revenue analysts can use expected values for many required inputs  
29 and prepare a P50 forecast. They can then apply the procedures outlined here to determine the  
30 probabilities of attaining other levels of demand or revenue.

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## 35 BACKGROUND

36 Concerns about the reliability of travel demand forecasts have been raised in a number of past studies.  
37 The seminal, and often-cited, work of Pickrell (2) in the early 1990s focused attention on the large  
38 differences between transit ridership forecasts that were developed for proposed new transit systems using  
39 travel demand models and the actual ridership levels that were experienced after those systems were built.

1 His work indicated that the models' ridership forecasts on average not only deviated significantly from  
2 actual ridership but that the forecasts were systematically higher than actual ridership by a significant  
3 margin. Other researchers have found similar effects both in the U.S. and internationally. Flyvbjerg (3)  
4 reviewed a wider range of public works projects including both transit and toll road projects and found  
5 both large and systematic gaps between model forecasts and actual usage. Bain's work (see, for example,  
6 6) focused on toll roads mirrors these other observations. Lemp and Kockelman (5) provide a  
7 comprehensive review of risks and uncertainties in forecasting demand for toll road projects.

8 This past work has identified a number of reasons why forecasts are often quite different from actual  
9 experience. These reasons generally fall into three categories:  
10

- 11 1) **Model structure and data** – Travel demand forecasting models are at best simplifications  
12 of the decisions that travelers make in response to transportation services and of the  
13 response of the transportation system to those decisions (e.g., volume/delay relationships).  
14 And, the data that are used to represent both the activity systems that generate demand and  
15 the transportation system that carries that demand generally do not completely  
16 characterize all of the details of those systems. While there has been considerable work  
17 over the past several decades to improve these models and data, there is still much work  
18 remaining to be done to address known limitations (see, for example, 6).
- 19 2) **Analysis bias** – The previously-cited studies of Pickrell, Flyvbjerg and Bain conclude that  
20 the systematic over-prediction of travel demand for transit, toll road and other projects can  
21 be explained in large part by a tendency of modelers to adopt overly-optimistic assumptions  
22 with respect to the projects that they are evaluating. These can be attributed to factors  
23 ranging from client pressure and deliberate misrepresentation of model results to a  
24 sometimes unconscious "optimism bias".
- 25 3) **Inherent uncertainties in future conditions** – Travel demand forecasting models include  
26 projections of details of the study area's activity and transportation systems into the future.  
27 For example, population and employment projections are required at a relatively fine level  
28 of geography – at a much higher granularity and often for a much longer time frame than  
29 the U. S. Census or other commercial population and employment forecasting firms provide.  
30 These projections can have significant effects on travel demand forecasts and yet we know  
31 that the projections themselves are inherently uncertain, affected by factors such as  
32 economic cycles which have similar associated uncertainties.

33 As a result of all these factors, it simply is not realistic to assume a single point forecast of travel  
34 demand will be sufficient to make informative planning decisions where the magnitude of a demand  
35 forecast determines if key objectives will be achieved. The factors above are related to uncertainties in the  
36 direct inputs to the models, the structure of the models and the values of the model parameters, all of  
37 which need to be considered in determining the levels of uncertainty associated with model outputs.

38 The need to formally consider uncertainties in forecasts has been recognized in the literature by many  
39 others and quantitative methods for representing the uncertainties have been used in selected applications  
40 such as energy consumption forecasting since at least the early 1970s (see, for example, 7). Lewis

1 provides a very articulate call for application of risk analysis in transportation planning and provides  
2 examples of how it can be applied to cost estimation. (8)

3 While there have been only limited applications of comprehensive quantitative risk analysis to travel  
4 demand forecasting, uncertainties in the forecasts have been represented in other ways. The most common  
5 approach is to conduct sensitivity analysis in which key inputs and model assumptions are varied  
6 individually and the full travel demand model is run to determine their effects on the forecast metrics of  
7 interest. Sensitivity testing is routinely conducted with travel demand models and, for example, is  
8 generally considered to be a key component of “investment grade” forecasting exercises for toll facilities.  
9 These analyses provide valuable information about the models’ responses to key inputs and the potential  
10 variability of forecasts. However, the levels of each variable tested are typically arbitrarily set and do not  
11 necessarily correspond to any particular likelihood of occurrence. And, more importantly, sensitivity  
12 analyses are typically conducted by varying only one variable at a time and thus interactions among these  
13 variables are not represented.

14 Scenario testing, in which likely future conditions are described by a combined set of changes in  
15 model inputs, is commonly used to test the interactive effects among two or more uncertain inputs. Often,  
16 these are used to test a set of “optimistic” (upside) or “pessimistic” (downside) assumptions and they can  
17 provide useful information about outcomes ranging from expected to extreme. They can also provide  
18 some indication of risk and, in fact, are commonly used as stress tests in financial modeling. However,  
19 running a small number of discrete scenarios cannot fully describe the range of outcomes or the relative  
20 likelihoods of those outcomes and thus should not be considered a complete substitute for a full  
21 quantitative risk analysis.

22 In concept, a full quantitative risk analysis involves a relatively straightforward two-step process:

- 23 1) **Estimate the probability distributions of key model inputs and assumptions** – There are  
24 many model inputs and assumptions that could significantly affect model forecasts. These could  
25 include, for example: land use patterns/distribution, employment, fuel cost and  
26 environmental/regulatory constraints on future development. In some cases the modeler has data  
27 that can be used to estimate both the shapes and parameters of the distributions of the key drivers.  
28 For example, if survey data are used to estimate model parameters, the sampling error  
29 distributions can be used for those parameters. Some state forecasting research agencies and  
30 commercial firms that produce regional population and employment projections recognize these  
31 uncertainties by publishing data on the magnitudes of their forecast errors over different time  
32 horizons. Those data can similarly be used to estimate the probability distributions of those model  
33 inputs. However, there are inevitably model inputs for which there are no data available to  
34 describe likely ranges or distributions. For these cases, either the project team’s  
35 subjective/professional judgment can be used to estimate distributions or a more formal approach  
36 such as the Delphi Method (9) can be used. In either case, the assumptions are at least then  
37 explicit and their effects on risk can be formally tested.
- 38 2) **Estimate the probability distributions of the model outputs** – Given the probability  
39 distributions of the inputs and a model that uses those inputs to generate outputs, the distributions  
40 of those outputs can be determined either analytically or, more commonly, using Monte Carlo  
41 simulation. An analytical calculation is possible if the input distributions are relatively simple  
42 (e.g., all independent uniform) and the model can be expressed in a relatively simple closed-form  
43 (e.g., linear). However, these conditions are not generally met with travel demand forecasting  
44 models and so some form of Monte Carlo simulation is typically required. Fortunately, excellent,

1 easy-to-use commercial and open-source packages are available to conduct Monte Carlo  
2 simulation.

3 *While this process is generally straightforward and has been applied to many other applications, its*  
4 *direct application to typical travel demand forecasting models is computationally intractable.* This is  
5 because Monte Carlo simulations typically require thousands of random draws of the inputs to reliably  
6 estimate output distributions, each requiring the model to be run to generate the outputs that correspond to  
7 the inputs. Most regional travel demand forecasting models require hours or days to run and even a very  
8 simple model that could run in minutes would require a significant Monte Carlo simulation runtime. Even  
9 though computation speeds have been improving with faster computers and distributed processing, travel  
10 demand models have increased in geographic resolution and complexity at rates that have offset these  
11 computational speed improvements.

12 In addition to the approach that is described in more detail in this paper, at least four other approaches  
13 have been used to support quantitative risk analyses of travel demand model forecasts.  
14

15 1) **Treating the inputs as independent effects** – Simple univariate sensitivity tests with the  
16 full travel demand forecasting model can be used to estimate how each variable individually  
17 affects the outputs. The differences from base outputs can be represented as simple  
18 deviations for each input and these differences, summed across all inputs, can be used in a  
19 Monte Carlo simulation to represent forecasts for the combinations of changed inputs  
20 represented by each set of Monte Carlo draws. The limitation of this approach is that it does  
21 not address the often complex interactions among inputs that can make their combined  
22 effects much different from the simple sum of their individual effects.

23 2) **Bayesian melding** – Sevcikova, Raftery and Waddell (10) used Bayesian melding with  
24 Urbansim to efficiently generate posterior distributions of the outputs based on prior  
25 distributions of inputs and data on outputs for selected years of the simulation. This  
26 approach works well for this type of application in which data are available on outputs to  
27 support the estimation of weights for the melding process.

28 3) **Using a “structure and logic” version of the model** – HDR and a predecessor firm, HLB,  
29 have used an approach in which they create a separate, simplified, travel demand  
30 forecasting model that represents the basic structure of the process that generates travel  
31 demand, without the full complexity of a complete regional travel model (11). This model  
32 can then be embedded in a Monte Carlo simulation to generate the distributions of outputs.

33 4) **Simplifying the travel demand model** – Zhao and Kockelman (12) were able to conduct  
34 100s of runs to determine how errors propagated through four-step models by creating a  
35 faster-running but otherwise equivalent version of the travel demand model that they were  
36 using. They also noted that “... *simple regressions of outputs on inputs offer very high*  
37 *predictive power, suggesting that prime sources of forecast uncertainties can be rather quickly*  
38 *deduced – and exploited – for better prediction.*”

39 All of these approaches represent ways of quantifying the likelihood of different model outcomes  
40 given uncertainties in the model structure, parameters and inputs. The first two listed are relatively

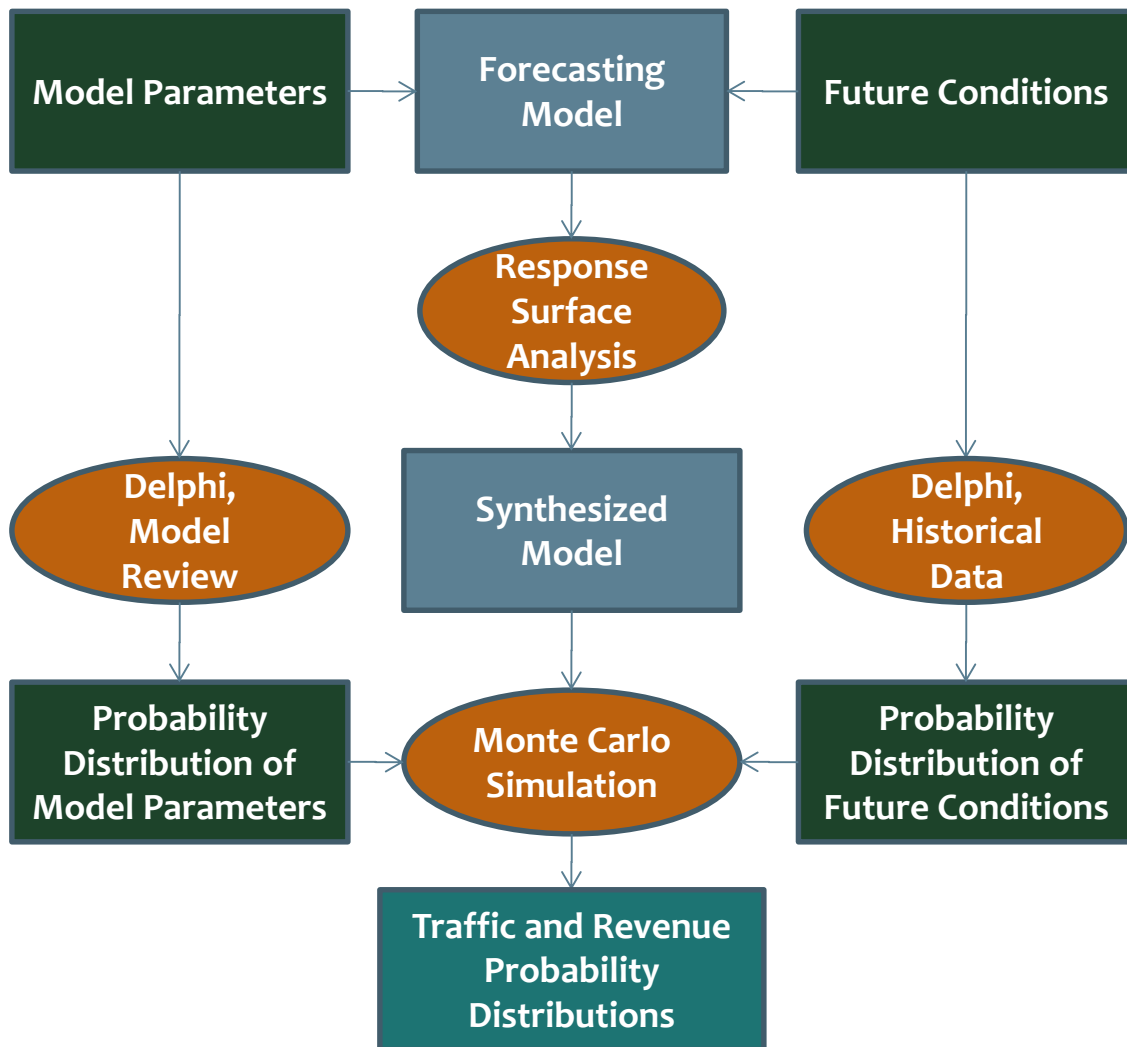
1 straightforward and can be done with little additional effort, though Bayesian melding does require  
2 longitudinal data that are not commonly compiled for travel demand models. The second two approaches  
3 both require additional modeling work and potentially a fair amount of effort; creating and validating a  
4 new parallel “structure and logic model” or creating a more computationally-efficient version of the  
5 original travel demand, retaining its structure.

6 The approach that is detailed in this paper follows from observations similar to those of Zhao and  
7 Kockelman; most notably, that closed-form models can be created using multivariate statistical analysis  
8 that very closely approximates the behavior of the much more complex and computationally demanding  
9 regional travel demand forecasting models. The need to create an analytically-tractable representation of a  
10 complex process is common in laboratory sciences and engineering and response surface methods have  
11 been developed to efficiently and effectively fill that need (13).

12 Response surface modeling uses statistically-efficient experiments and a variety of statistical methods  
13 are used to create models describing the relationships between process (model) inputs and outputs. In this  
14 application, the experiments are simply combinations of travel demand model inputs, constructed in ways  
15 that support the statistical estimation of models which describe the relationship between the original  
16 model’s inputs and outputs. Note that these resulting “synthesized” models are not equivalent to  
17 “structure and logic” models in that: a) they are direct statistical derivatives of the full travel demand  
18 models in respect to the relationships between selected inputs and outputs and b) they cannot be created  
19 without first having a full travel demand model.

20 Figure 1 shows conceptually how response surface analysis can be used to support quantitative risk  
21 analysis around travel demand forecasts.

1 **Figure 1 - Quantitative Risk Analysis Using Response Surface Methods**



2

3 In this schematic, the forecasting model is a regional travel demand model or whatever model is used  
 4 to create the desired travel demand forecasts. There are uncertainties in the model parameters and  
 5 structure as indicated in the boxes on the left side of this figure and uncertainties in inputs such as future  
 6 conditions and other data as indicated on the right side of the diagram. Probability distributions can be  
 7 developed for all of these uncertain factors using the approaches described previously. The travel demand  
 8 forecasting model can be “synthesized” using response surface methods into an analytically-tractable  
 9 model that can then be imbedded in a Monte Carlo simulation to generate distributions of the outputs of  
 10 interest.

11 The following section describes one of the applications of this approach to travel demand forecasting  
 12 that authors of this paper have completed over the past several years.

### 13 **THE ORLANDO I-4 EXPRESS LANES APPLICATION**

14 Florida’s Interstate 4 connects the Tampa Bay and Daytona regions through the Orlando region in  
 15 Central Florida. Several factors, including the significant growth of the Orlando region, have led to  
 16 correspondingly high growth in I-4 traffic through the Orlando region. And, that growth has led to a need



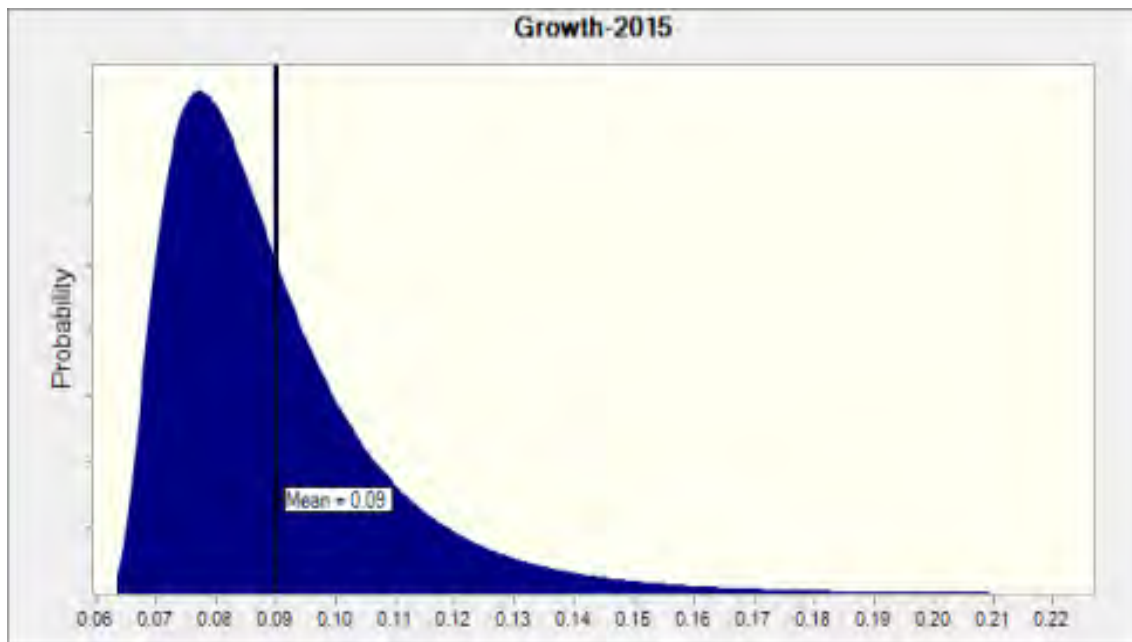


1 growth rates, estimated values of time, completion of other network capacity enhancements and toll rates  
 2 were key drivers of the model's forecasts. The first three of these represent uncertain model inputs and  
 3 toll rates are policy variables that can be controlled by the facility owner/operator.

4 Because toll revenues will provide an important part of the project's funding, traffic and revenue  
 5 estimates were desired to represent levels that would be met or exceeded with a probability of 0.75. This  
 6 corresponds with a relatively risk-averse position, providing some assurance that the projected revenues  
 7 would most likely be realized, as compared to more typical "expected values" which would theoretically  
 8 represent over-estimates 50% of the time.

9 The first step of the process was to develop probability distributions for each of the three key  
 10 uncertain inputs. Projected population growth rates for the Orlando area and Florida as a whole changed  
 11 significantly after the most recent recession. Baseline population forecasts for the region were derived  
 12 from projections developed by the Bureau of Economic and Business Research (BEBR) at the University  
 13 of Florida. BEBR has several decades of experience producing forecasts for Florida counties and has  
 14 published reports described its forecasting accuracy over different projection horizons and also considered  
 15 the recent economic downturn with forecast adjustments. This information was used to estimate  
 16 probability distributions around the current estimates for each of the forecast years. Figure 3 shows an  
 17 example study area population growth probability distribution through the year 2015.

18 **Figure 3 - Example Input Distribution for Population Growth to 2015 (fractional increase over 2010)**



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20 Values of time were estimated using data from travel surveys and the associated sampling error  
 21 distributions were used to provide probability distributions for these estimates. Finally, the likelihoods of  
 22 completing various other network capacity enhancements were subjectively estimated based on input  
 23 from Florida DOT planning engineers.

24 The response surface modeling used a relatively simple fractional factorial experimental design for  
 25 the travel demand model runs. Nine different run conditions were specified, run for each of seven forecast  
 26 years (five-year increments through 2045). Figure 4 shows these nine conditions (14).

1 **Figure 4 - I-4 Managed Lanes Study Experimental Design**

Alternative	VOT/hour	Economy	Network	Toll Rate/Mile
P1	\$10.67	BEBR Med-Low	E+C	\$0.05
P2	\$10.67	BEBR Med	150	\$0.15
P3	\$10.67	BEBR Med-High	125	\$0.10
P4	\$18.08	BEBR Med-Low	125	\$0.10
P5	\$18.08	BEBR Med	150	\$0.05
P6	\$18.08	BEBR Med-High	E+C	\$0.15
P7	\$25.49	BEBR Med-Low	125	\$0.15
P8	\$25.49	BEBR Med	E+C	\$0.10
P9	\$25.49	BEBR Med-High	150	\$0.05

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3 Value of time was varied in three levels and the economy (population growth) was similarly varied  
4 with three levels, representing BEBR's medium-high, medium and medium-low projections. Completion  
5 of other roads in the Orlando network, some of which could divert traffic from the facility and others of  
6 which could provide feeder capacity, was represented by existing plus committed project (E+C), those  
7 projects needed to keep all major roads in the network at a volume to capacity ratio below 1.5 (150) and  
8 those necessary for volume capacity ratios below 1.25 (125). Finally, daily average toll rates were varied  
9 in the regional model between \$0.05/mile and \$0.15/mile.

10 The response surface modeling used data from these runs to determine the functional form that best  
11 fit the observed model response patterns. The resulting models are nonlinear in the variables but linear-in-  
12 the-parameters and so could be easily estimated with standard statistical software. As observed by Zhao  
13 and Kockelman in their work, these simple statistical models fit the data very closely ( $R^2 \sim 0.98$ ),  
14 suggesting that the synthesized models capture the original travel demand model's behavior very closely  
15 in these dimensions. Figure 5 shows an example model developed for I-4 traffic volumes; similar models  
16 were also generated for toll revenues (14).

1 **Figure 5 - Example I-4 Response Surface Model**

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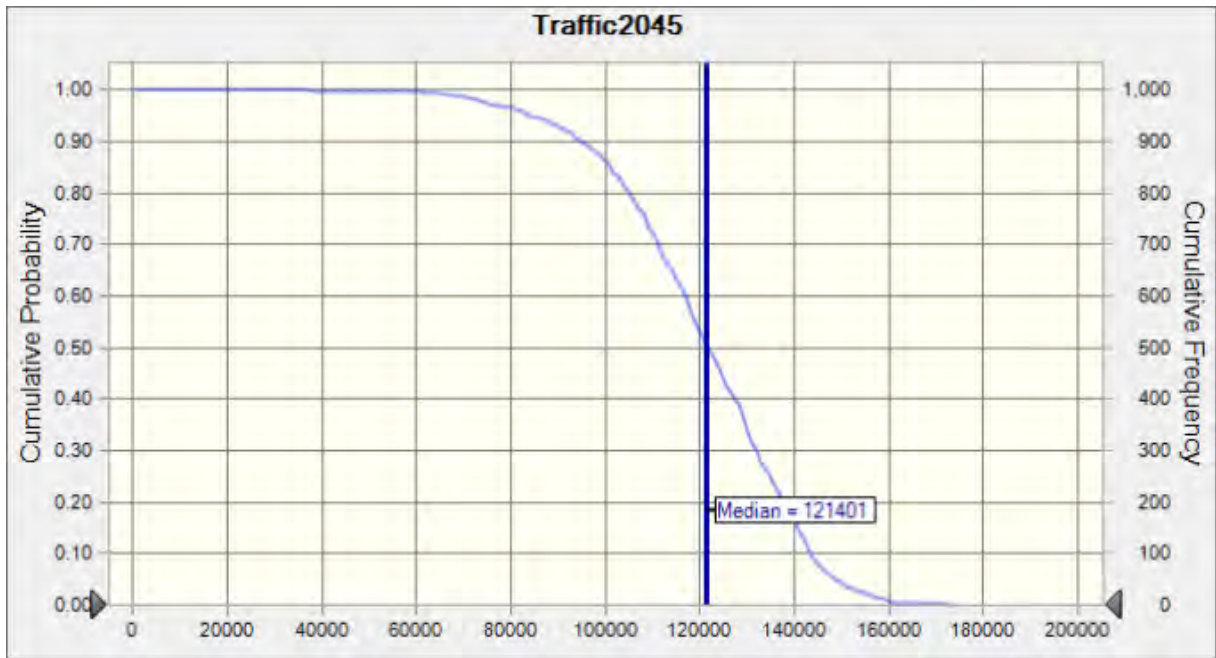
$$Traffic = 128686 + 48486 * growth - 91138245 * (tollRate / VOT) + 9432 * \ln(rampUp) - 11099 * roadEC - 15820 * road150 + yearCon$$

Where: *Traffic* is the number of daily one-way trips that use the I-4 Express Lanes  
*growth* is the ratio of dwelling units in the given year to dwelling units in 2010 minus one  
*tollRate* is the average toll rate charged on I-4 in 2010 \$  
*rampUp* is the number of years the project has been operating in the given year  
*roadEC* represents the road improvements included only in the E+C conditions  
*road150* represents an improvement program that maintains all roads below V/C of 1.5  
*yearCon* is a vector of constants representing the years for which the forecasts are being made

3

4 The response surface models were programmed along with the estimated distributions of the inputs in  
 5 a Monte Carlo simulation. The simulation was run with between 100,000 and 1,000,000 draws to estimate  
 6 the distributions of selected outputs. Figure 6 shows the reverse cumulative distribution output from one  
 7 of the simulation runs.

8 **Figure 6 - Example I-4 Traffic Distribution (average daily traffic)**



9

10 Models were developed for each of the project’s build scenarios and the simulations were used to  
 11 develop traffic and revenue distributions. These distributions showed the requested 75<sup>th</sup> percentile values,  
 12 along with all other percentile values.

13

## 1 CONCLUSIONS AND RECOMMENDATIONS

2 Travel demand forecasting models are not now, and never will be, perfect representations of the  
3 systems that they represent and so there are inevitably uncertainties around the forecasts that these models  
4 generate. The uncertainties derive from model structure, parameter estimates associated with the structure  
5 and data including forecasts of inputs to the models. Many if not most of these uncertainties are inherent  
6 in the modeling process and create risks associated with projects or programs which are supported by the  
7 models' forecasts. So, it is important to recognize those uncertainties in some way. Past studies have used  
8 a variety of approaches to represent the uncertainties and risks, including simple sensitivity and scenario  
9 analyses. However, those approaches do not fully quantify the levels of uncertainty in the forecasts.

10 There have been some applications of more formal quantitative risk assessment for travel demand and  
11 land use forecasting models, also using a variety of approaches. This paper details one approach that can  
12 be used with any travel demand forecasting model system, that does not require any calibration data  
13 beyond that used for the development of the original travel demand model system and that directly  
14 mirrors the behavior of that system while being computationally tractable when imbedded in a Monte  
15 Carlo process. The formal risk analysis approach described here can assist by providing a more complete  
16 evaluation of a project's likelihood of achieving specified objectives. In addition, it can have a broader  
17 application in the development of traffic and revenue forecasts other than a "most likely" or 50%  
18 probability of attainment ("P50") scenario.

19 The prevalence of uncertainties in travel demand forecasting, the risks that can be presented by those  
20 uncertainties and the availability of approaches for quantifying those risks together present a compelling  
21 case for more frequent applications of formal risk analysis in travel demand forecasting.  
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